

Mathematical Sciences Colloquium Series

Friday (April 4th, 2014) 3:00pm Room 103A

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On the Crystal Basis Theory of $\widehat{\mathfrak{sl}}_2$

The Lie algebra consisting of two by two complex matrices of trace zero, denoted $\mathfrak{sl}_2(\mathbb{C})$, is considered the building block of all Lie algebra representation theory. The *quantum group* for $\mathfrak{sl}_2(\mathbb{C})$ is a deformation of its universal enveloping algebra, created by incorporating a quantum parameter q . Taking the limit as q approaches one, called the classical limit for a representation of the quantum group, produces the corresponding representation for the universal enveloping algebra. This, in turn, corresponds to a representation of $\mathfrak{sl}_2(\mathbb{C})$. Crystal bases for representations of the quantum group can be thought of as bases for these representations when $q = 0$, the corresponding limit is called the crystal limit. While they may seem complicated at first, crystal bases exhibit numerous combinatorial patterns. As an example of this, the Tensor Product Rule for finite Lie algebras will be briefly described. The talk will conclude with preliminary, expository result from $\widehat{\mathfrak{sl}}_2(\mathbb{C})$ the affine version of $\mathfrak{sl}_2(\mathbb{C})$, and a new possible combinatorial representation of highest weight vectors in $\widehat{\mathfrak{sl}}_2(\mathbb{C})$.